Transportation Theory and Planning of a School Schedule

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1 Introduction

The theory of planning a school schedule requires to fit the classes, their teachers or lecturers, the students (members), places (rooms) and the time periods (hours) so as not to cause a collision, i.e. no two classes are allowed to be conducted in the same place and time, and no participants are allowed to have classes in a given moment in more than one place.

The Transportation Theory optimizes the method of transporting goods to minimize the time or the cost of the transport (or, equivalently, to maximize one's profit).

The goal of this work is to present a method for converting the school schedule planning theory to a Transportation Problem.

2 The conversion

Let:

- S - the set of rooms,
- G - the set of students (or student groups),
- P - the set of teachers,
- C - the set of possible starting times.

The resources that need to be split are the rooms coupled with the class starting hours, and the receivers are the teachers coupled with the student groups. Thus, the set of producers is $S \times C$, and the set of receivers - $P \times G$.

Let:

- the number of producers $m = \overline{\overline{S \times C}}$,
- the number of receivers $n = \overline{\overline{P \times G}}$,
- the quantity of production $A_i = 1 \qquad i = 1, 2, \ldots, m$ (each room/hour pair can be used by just one class),
- the quantity of demand $B_j = 1 \qquad j = 1, 2, \ldots, n$ (each teacher/group pair can have at most one class at a time),
- the cost of assigning classes to rooms be constant, e.g.

$$c_{ij} = \begin{cases} 1, & i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \\ 0, & j = n + 1 \qquad \text{(in case a storage exists)} \end{cases}$$

If $m < n$ (the number of possible starting times is less than the number of classes), the problem has no solution.

If $m > n$ (more production than demand - an open Transportation Problem), you need to introduce a fake receiver (a "storage"), whose demand shall equal the difference: $B_{n+1} = m - n$, allowing us to have a closed Transportation Problem.

When creating the cost function, you need to take the following conditions into account:

1. any $(s, c) \in S \times C$ pair cannot be assigned to more than one receiver,
2. any $(p, g) \in P \times G$ pair cannot be assigned to more than one producer (the only receiver with more than one producer can be the additional fake receiver - the storage),
3. a student group $g \in G$ cannot be assigned to more than one producer with a given class starting time,
4. a teacher $p \in P$ cannot be assigned to more than one producer with a given class starting time.

The cost function shall be constructed the following way:

$$f(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} + O_i M + K_i M + L_i M + N_i M) x_{ij}$$

or, when the storage exists:

$$f(X) = \sum_{i=1}^{m} \sum_{j=1}^{n+1} (c_{ij} + O_i M + K_i M + L_i M + N_i M) x_{ij}$$

where:

- $X = [x_{ij}]_{i=1,2,\ldots,m;j=1,2,\ldots,n}$ or $X = [x_{ij}]_{i=1,2,\ldots,m;j=1,2,\ldots,n+1}$ (when the storage exists) - the matrix of $(s,c) \in S \times C$ pairs assigned to receivers (meaning the amount of transport from the producer to the receiver),

- O_i - a coefficient for finding class collisions:

$$O_i = \sum_{j=1, j\neq i}^{m} x_{ij}$$

- N_i - a coefficient for finding time and room collisions:

$$N_j = \sum_{i=1, i\neq j}^{n} x_{ij}$$

- K_i - a coefficient for finding teacher collisions:

$$K_i = \sum_{(p_i, g)\in P\times G} \sum_{j=1, j\neq i}^{m} x_{ij}$$

  where p_i means a teacher with a class in the i-th element of the $P \times G$ set.

- L_i - a coefficient for finding student collisions:

$$L_i = \sum_{(p, g_i)\in P\times G} \sum_{j=1, j\neq i}^{m} x_{ij}$$

  where g_i is a student group in the i-th element of the $P \times G$ set.

- M - a number high enough for the cost function to raise significantly when two or more (s,c) pairs are assigned to one receiver, e.g. $M = (m+n+1)$, but it can also be a constant.

Constructing the cost function this way (along with limiting the amount of production and demand) causes results which have just one (s,c) pair assigned to one receiver (except for the optional storage).

3 Conclusions

The problem of planning a school schedule can be converted to a non-linear closed Transportation Problem.